Modern Structural Analysis - Introduction to Modelling

Supplementary information

Chapter 3 Section 5.10 Equivalent beam model for parallel chord trusses

The cross references in the form 'n.m' or 'n.m.p' are to sub-sections in *Modern Structural Analysis - Introduction to Modelling.*
The cross references with a single number are to items within this document.

Case study 1 - Checking model for a plane parallel chord truss

Purpose
The purpose of this study is to investigate:

- an example of the use of the equivalent beam checking model for a plane parallel chord truss - Section 5.10
- the behaviour of a truss of this type

Truss configuration
Figure 1 shows a plane truss with parallel chords fabricated from circular hollow sections (CHS).

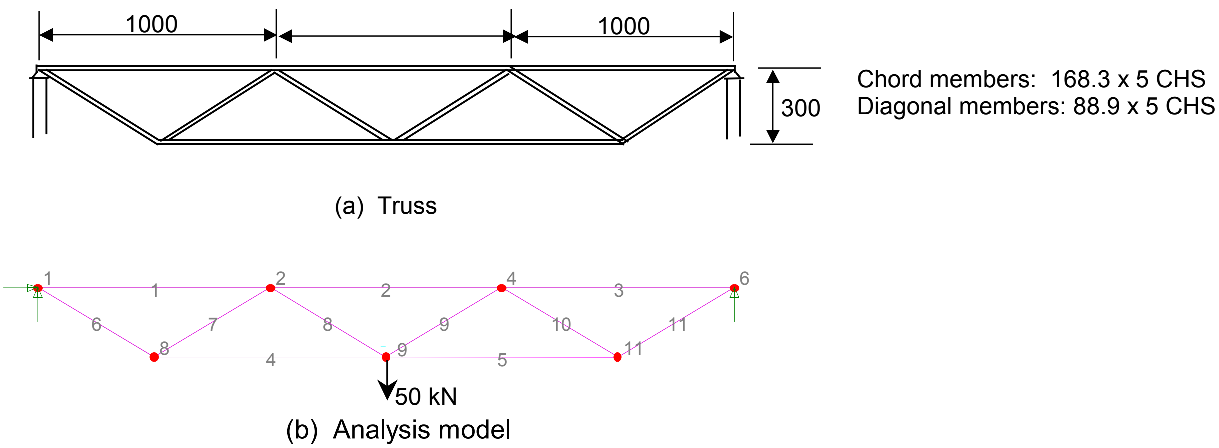


Figure 1 In-plane parallel chord truss

Element model

Figure 1(b) shows the 2D element model.

Model types Two types of model are considered:

- *Beam element model* which includes axial, bending (but not shear) deformation. Full moment connections at all joints
- *Bar element model* which includes only axial deformation of the members

Element Properties - see Table 1

Table 1 Element properties

Member	**Area** (mm^2)	**I value** (mm^4)	E (kN/mm^2)
Chords	2570	8.56E+08	209
Diagonals	1320	1.16E+06	209

The *supports* are:

- Node 1 - restrained in global x and y directions; no rotational restraints
- Nodes 6 - restrained in the vertical direction only.

Loading 50 kN vertical checking load at the centre of the span applied at the lower chord level.

Calculate the central deflection in the line of the applied load - Δ - using the equivalent beam of Section 5.10.4

For a definition of symbols see Section 5.10.4.

The equivalent beam is shown in Figure 2

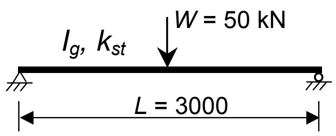


Figure 2 Equivalent beam

Section properties for the equivalent truss

For I_g use Equation (5.17)

$I_g = A_c b^2 =$ 2570*300^2/2 = 1.157E8 mm^4

For the shear stiffness K_{st} use Equation (5.19) (there are no posts)

$\theta = \tan^{-1}(300/500) =$ 0.5404

$K_{st} = fE_d A_d \sin^2\theta \cos\theta =$ 1.0*1320*209*sin(0.5404)^2*cos(0.5404) =62617 kN/rad

Central deflection of the equivalent beam

$\Delta = \Delta_b + \Delta_s$ (Equation 5.5)

$= WL^3/(48EI_e) + WL/(4K_{st})$ (From Table A4)

= 50*3000^3/(48*209*1.157E8) + 50*3000/(4*62617)

= 1.163 + 0.5989

= 1.762 mm

Comparison of values of central deflection with element model results
Table 2 compares the results for central deflection for the equivalent beam model, the beam element model and the bar element model.

Table 2 Comparison of Δ values

	1	2	3	4	5
Displacement	**Equivalent beam**	**Beam element model**	**%diff** (1-2)/2*100	**Bar element model**	**%diff** (1-4)/4*100
Δ_b - axial, chords	1.164	0.814	42.88	1.228	-5.26
Δ_s - axial, diagonals	0.599	0.382	56.70	0.599	0.01
Δ_m - bending		0.278			
Δ	1.762	1.475	19.49	1.827	-3.54

Note:

- Δ_b is the contribution of the axial deformation of the chords to the central deflection. For the equivalent beam this is the central deflection due to bending mode deformation.
- Δ_s is the contribution of the axial deformation of the diagonals to the central deflection. For the equivalent beam this is the central deflection due to shear mode deformation.
- Δ_m is the contribution to Δ from the moments in the beam element model i.e. from bending in the elements of the beam element model.

Calculate the contribution of axial and bending deformation in the beam element model using the principle of virtual work
Table 3 shows the calculation of the contributions to Δ from axial deformation in the beam element model.

The columns of Table 3 represent:

- N_a - the actual axial forces in the elements from the LUSAS output
- N_v - the virtual forces in the elements due to a unit point load in the direction of the required displacement. In this case the required displacement is in the line of the applied load and is therefore the N_a column divided by 50. For displacements in an other direction a separate load case with a unit load in that direction would be required to establish the N_v column.
- A and L are the areas and the lengths of the members respectively.

Table 3 Virtual work calculation of the deformation of the beam element model

element	N_a	N_v	A	L	$N_aN_vL/(EA)$	
					chords	diagonals
1	-38.03	-0.7606	2570	1000	0.0539	
2	-98.88	-1.9776	2570	1000	0.3641	
3	-38.03	-0.7606	2570	1000	0.0539	
4	67.83	1.3566	2570	1000	0.1713	
5	67.83	1.3566	2570	1000	0.1713	
6	44.14	0.8828	1320	583.10		0.0824
7	-34.64	-0.6928	1320	583.10		0.0507

8	37.04	0.7408	1320	583.10		0.0580
9	37.04	0.7408	1320	583.10		0.0580
10	-34.64	-0.6928	1320	583.10		0.0507
11	44.14	0.8828	1320	583.10		0.0824
				Sum	0.8144	0.3822

The sums of the last two columns in the table give the contributions to Δ of the chord and the diagonal elements of the model. The contribution of bending deformation - Δ_m = 1.475 - 0.814 -0.382 = 0.278 mm) is the difference between the total deflection and the sum of these two axial components

Calculate the contribution of chords and the diagonals in the bar element model
The same process as for the beam element model was repeated for the bar element model to calculate the contributions of the axial deformations of the chords and of the diagonals to Δ The results are quoted in Table 3 (calculation not shown here).

Analysis of the results in Table 3
The following trends are identified:

- The dominant contribution to Δ is the axial deformation of the chords. The bending deformation of the elements has the least effect but it is not negligible in the beam element model.
- The equivalent beam model significantly overestimates the central deflection by 19% as compared with the beam element model results due to the fact that it does not take account of the bending of the elements.
- The equivalent beam model gives very good correlation with the results from the bar element model in this case.

Estimate the values of the axial forces in the members of the frame

See last part of Section 5.10.4

Table 4 Axial forces in the plane truss

	1	2	3	4	5
Axial force	**Equivalent beam**	**Beam element model**	**%diff** (1-2)/2*100	**Bar element model**	**%diff** (1-4)/4*100
N_c Top chord	125.0	98.87	26.4	125.0	0.0
N_c Bottom chord	125.0	67.83	84.3	83.33	50.01
N_d End diagonal	48.59	41.76	16.4	48.59	0.0

Chord members For the equivalent beam model the axial forces in the chords are calculated using:

$N_c = M/b$

where:

- N_c is the axial force in a chord member at the position where M is calculated
- M is the (central) bending moment in the equivalent beam.
- b is the distance between the chord members

For the axial force at the centre of the span
$N_c = M/b = WL/4/b = 50*3000/4/300 = 125.0$ kN
Table 4 compares this result with the values from the beam and bar element models.

End diagonal members
The axial force in the end diagonal is estimated by resolving the forces at the support - Figure 9.
Vertical equilibrium gives $25.0 = N_d \sin(\theta)$ hence $N_d = 25.0/\sin(0.5404) = 48.59$ kN

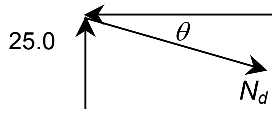


Figure 3 Forces at support

This result is also included in Table 3.

Analysis of the results of Table 3
The following trends are identified:
- The top chord axial force from the equivalent beam (125.0) correlates precisely with that from the bar element model but the bottom cord axial forces are lower because of the diagonals that connect to it at the loaded node.
- The top chord value for the beam element model is significantly less due to the bending stiffness of the elements.
- The end diagonal forces from the equivalent beam and the bar element model precisely correlate. The effect of element bending on the end diagonal axial force is less pronounced.

General conclusions
The results of this case study indicate the type of correlation that can be achieved in using the equivalent beam model. It is recommended that these results are not extrapolated to other truss configurations without cross checking against element model results.

Case study 2 - Checking model for a parallel chord truss with a triangular cross section

The purpose of this study is to investigate an example of the use of the equivalent beam checking model for a 3D parallel chord truss.

Truss configuration

Figure 4 shows a 3D truss with parallel chords fabricated from circular hollow sections (CHS). The configuration is similar to the plane truss of Case study 1 but there are two top chord members forming a triangular cross section for the system.

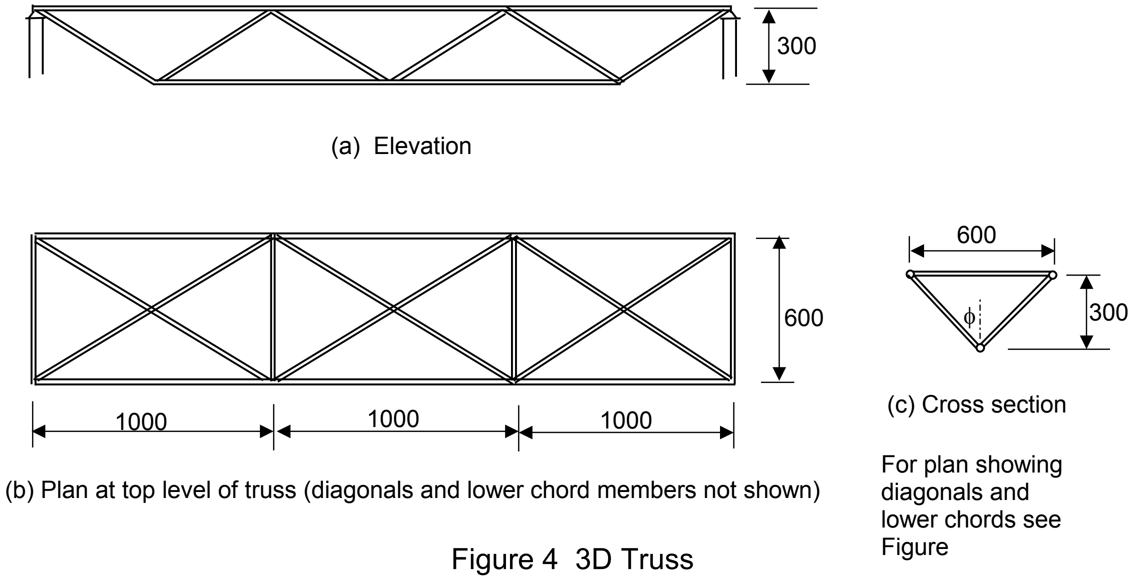


Figure 4 3D Truss

Element models

Figure 5 shows the 3D element model.
Beam and bar element models were used as for the truss of Case Study 1

The *element properties* are the same as for the truss of Case Study 1 (Table 1)

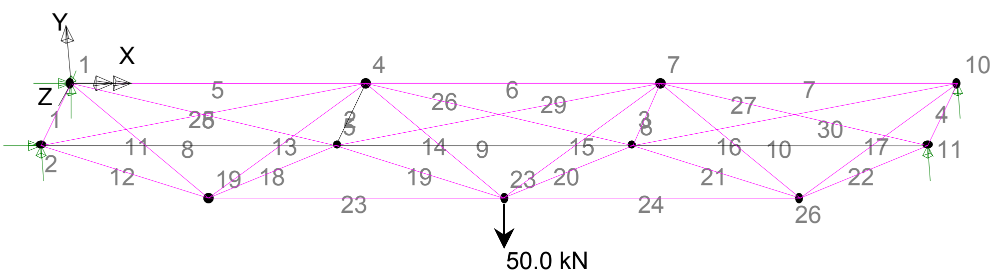


Figure 5 Analysis model for 3D truss

The *supports* are:

- Nodes 1 and 8 - restrained in global x,y and z direction; no rotational restraints
- Nodes 6 and 13 - restrained in the vertical direction only.

Loading 50 kN vertical checking load at the centre of the span applied at the lower chord level

Calculate the central deflection in the line of the applied load - Δ - using the equivalent beam of Section 5.10.4

For a definition of symbols see Section 5.10.4 .

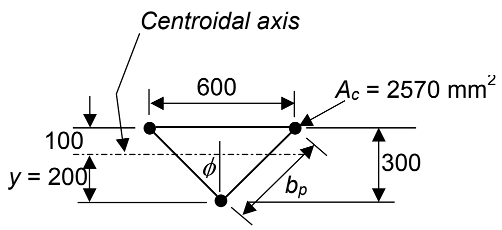


Figure 6 Cross section of equivalent beam

Equivalent I value I_g

Figure 6 shows the cross section for the equivalent beam . The three chord member areas form a 'section' for which the second moment of area is calculated by identifying the position of the centroidal axis of the section and taking second moments of area of the chord members about this axis.

Position of centroidal axis. Take moments of area about the lower chord member:
$3A_c y = 2A_c 300$ i.e. $y = 2*300/3 = 200$ mm
where:

- A_c is the area of a chord member
- y is the distance from the lower chord to centroidal axis

The second moment of are is therefore:
$I_g = A_c 200^2 + 2A_c 100^2 = 2570*200\text{^}2 + 2*2570*100\text{^}2 = 1.542\text{E}8 \text{ mm}^4$

Shear stiffness - K_{st}

To calculate the vertical shear stiffness of the truss, the shear stiffness in the inclined plane of the diagonals (i.e. the planar stiffness) is calculated and the vertical component of this is calculated (by a $\cos^2(\phi)$ transformation - see table A5) and then doubled to account for the two inclined planes.

The inclined *planar* trusses are oriented at an angle ϕ to the vertical - Figure 6.

The angle $\phi = \tan^{-1}(300/300) = 0.7854$ radians

Depth of a planar truss - $b_p =$ sqrt(300^2 + 300^2) = 424.3 mm
The θ angle for a planar truss (Figure 7) - $\theta_p = \tan^{-1}(424.26/500) = 0.7036$ radians

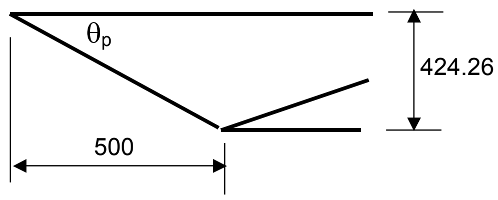


Figure 7 Angle θ_p

The shear stiffness for a planar truss $K_{st,p}$ is calculated using Equation (5.19):

$K_{st,p} = fE_dA_d\sin^2\theta_p\cos\theta_p$ = 1.0*209*1320*sin(0.70360)^2*cos(0.7036)
= 88051 kN/rad

The vertical component of the shear stiffness for the system is:

$K_{st} = 2\ K_{st,p}\cos^2(\phi)$ = 2*88051*cos(0.7854)^2 =88051 kN/rad

Value of Δ

$\Delta = \Delta_b + \Delta_s$ (Equation (5.5))
$= WL^3/(48EI_e) + WL/(4\ K_{st})$ Table A4
= 50*3000^3/(48*209*1.5428E8) + 50*3000/(4*88051)
= 0.872 + 0.426
= 1.298 mm

In Table 5 this value is compared with the corresponding results from the beam element and the bar element models

Table 5 Comparison of Δ values

Parameter		1 **Equivalent beam**	2 **Beam element model**	3 **%diff** (1-2)/2*100	4 **Bar element model**	5 **%diff** (1-4)/2*100
Deflection	Δ	1.298	1.019	27.38	1.231	5.4
Axial force	N_c Top chord	62.5	46.2	35.28	49.10	27.3
	N_c Bottom chord	125	65.06	92.13	83.33	50.01
	N_d End diagonal	27.32	24.3	12.43	27.32	0.0

49.10

Analysis of the results for deflection in Table 5

The equivalent beam gives good correlation against the bar element model results but is less accurate with the beam element model where bending deformation is not negligible.

Estimate the values of the axial forces in the members of the frame

Chord members

Treating the section of Figure 6 as that of a beam, the axial stress in the bottom chord member - σ_{cb} - is:

$\sigma_{cb} = My/I_e = M*200/I_g$

$M = WL/4 = 50*3000/4 = 37500$ kN mm

hence $\sigma_{cb} = 37500*200/1.542E8 = 0.04864$ kN/mm^2

The axial force in the bottom chord is therefore:

$N_{cb} = \sigma_{cb}A_c = 0.04864*2570 = 125.0$ kN

The axial force a top chord member is:

$N_{ct} = \sigma_{ct} A_c = MyA_cI_e = 37500*100/1.542E8*2570 = 62.50$ kN

where σ_{ct} is the axial stress in the top chord member

End diagonal member Vertical equilibrium at a support node:

$R = N_d\cos\lambda$

where:

- R is the reaction at the support = 12.5 kN
- N_d is the axial force in the diagonal member
- λ is the angle between the axis of the diagonal member at the support (e.g element 14 in Figure 5) and the vertical axis - See Figure 8

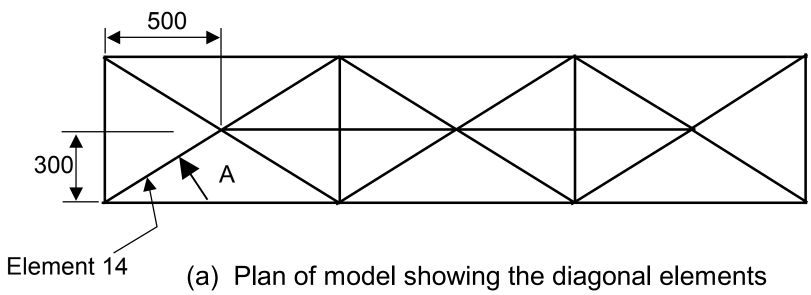
$$\cos\lambda = \frac{\text{projection of diagonal member on the vertical axis}}{\text{true length of the diagonal member}}$$

Length of diagonal member = sqrt(300^2 + 300^2 + 500^2) =655.74 mm

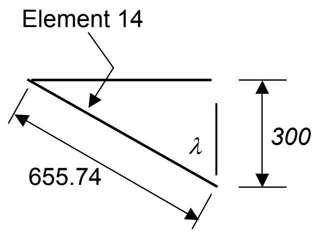
$\cos\lambda = 300/655.74 = 0.4575$

hence $N_d = R/\cos\lambda = 12.5/0.4575 = 27.32$ kN

The results for the three models are given in Table 5



(a) Plan of model showing the diagonal elements



(b) View on A

Figure 8 Definition of angle λ

Analysis of the results for axial forces in Table 5

The equivalent beam significantly overestimates the axial forces as compared with the beam element model. This is mainly due to the effect of element bending.

The correlation between the equivalent beam results and the bar element model results is good apart from that for the bottom chord. This is due to the effect of the diagonals meeting at the bottom central node.

A correlation with the bar element model for the bottom chord axial force is achieved by taking moments about the axis at 'a' on the free-body diagram of Figure 9. (a)
Applying the condition of equilibrium gives:

$25.0*100 = n_b*300$ hence $n_b = 25.0*1000/3000 = 83.33$

This is the same as the value from the bar element model