Supplementary information

Chapter 6 Section 6.3.4 Plate bending and shell element models

Case study 1 Rectangular concrete slab

Figure 1 (a) shows a plate bending element model for a rectangular slab under uniformly distributed vertical loading.

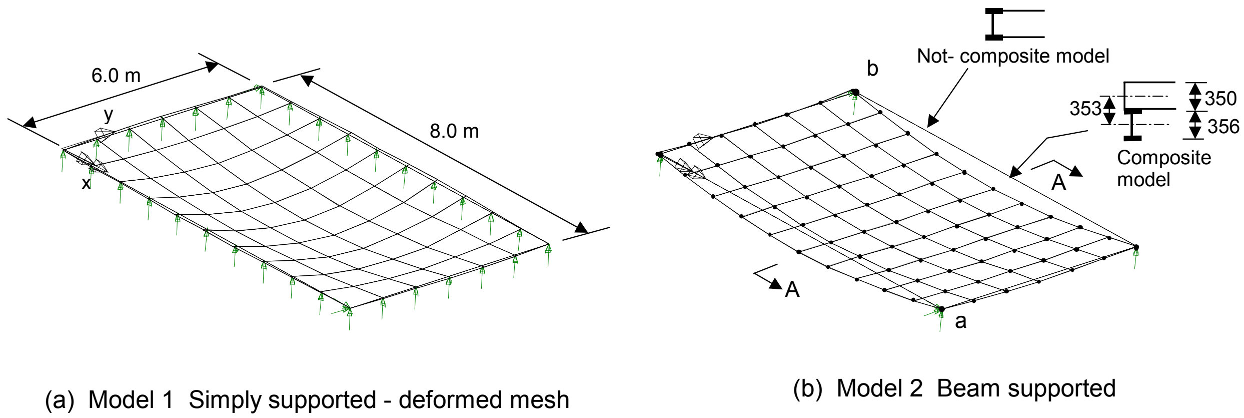

Figure 1 Rectangular slab models

Slab details:

Plan dimensions - 8.0 x 6.0 m; slab thickness - 350 mm;

E = 24 kN/mm^2; ν = 0.2

Indicative parameters:

- Δ - vertical deflection at centre of slab
- M_x - at centre of slab
- M_y - at centre of slab

Model 1 Simply supported

This is a plate bending model using LUSAS *Isoflex* 4 noded thin plate elements as shown in Figure 1(a). All sides of this model are simply supported as defined in Table 1.

Table 1 Restraints for simply supported model

Line	Translation Z	Rotation X	Rotation Y
Edge parallel to X axis	Fix	Free	Fix
Edge parallel to Y axis	Fix	Fix	Free

Results: The deformed mesh is shown in Figure 1(a). Note the 'dishing' effect due to two way spanning action. The main span is in the Y direction. The characteristic values to be considered are the vertical deflection (Δ) and the Mx and My moment at the centre of the slab. These are the maximum moments in this case. Results are given in Table 2

Table 2 Results for simply supported slab

	Timoshenko	FE model		Checking model	
		Value	% diff	Value	% diff
Δ	1.19E-03	1.16E-03	-2.66	1.968E-03	65.42
Mx	17.57	17.57	0.01		
My	31.75	32.17	1.32	45.0	41.72

All units kN and m. The % difference columns are calculated in relation to the Timoshenko solution.

The calculations for the Timoshenko solution is from Timoshenko and Woinosky-Krieger (1959), Table 8, page 120
The checking model is based on a 1.0 m wide strip spanning in the Y direction. The characteristic values for the strip are:

Width of strip b = 1.0, depth of slab $t = 0.35$

$I_{strip} = bt^3/12 = 1.0*0.35^3/12 = 3.573E\text{-}3\ m^4$

W = total load on strip = 10*6 = 60 kN, L = span of strip = 6.0 m

$\Delta = 5WL^3/(384EI) = 5*60*6^3/(384*24E6*3.573E\text{-}3)$

$M_y = WL/8 = 60*6/8 = 45.0$ kNm/m

Note that:

- The FE model and the Timoshenko results are close. The latter can be considered as an exact solution (Section 2.4.5)
- As expected, the checking model gives deflections and moments which are significantly larger than the FE values - of the order of 50% greater. Such a difference does not invalidate the strip model for checking. It gives deflections and moments which are significantly greater than from the FE model which is to be expected because it neglect the two-way spanning action.

Model 2 Beam supported

The slab of Figure 1(b) is modelled using 4 noded flat shell elements, corner columns (vertical restraints) and beam supports on all edges. Two sub-models are considered

1. Not composite model This has the basic properties as Figure 1(a) but:

- All sides have 356 x 171 x 67 kg/m UB beam supports.
- All corners have pin restraints i.e. restraints in the 3 translational directions.

The not composite model has no resultant internal force actions in the plane of the slab and gives the same results as a plate bending model.

2. Composite model This model is as for 'not composite' except that:

- A statically determinate set of restraints is applied in the plane of the slab so that they do not affect the internal force actions in the that plane. This is done by:
  - * All supports have vertical (z direction) restraints.
  - * Node 'a' is also restrained in the x and y directions
  - * Node 'b' is also restrained in the y direction.
- To take account of composite action between the slab and the beam, rigid links are inserted between the slab nodes and the corresponding beam nodes. This was implemented using an out of plane eccentricity that is included as data for the slab:

e = (0.356+0.350)/2 = 0.353 m

The relative positions of the slab and the beam for the non-composite and the composite models are shown in Figure 1(b).

The composite action in the composite model causes resultant in-plane stresses in the slab - Section 6.3.4. This is why flat shell elements are needed.

Results

Figure 1(b) shows the deformed mesh. Note how the main span is in the x direction rather than in the y direction as compared with the simply supported case of Figure 1(a). Results for the indicative parameters are given in Table 3.

Table 3 Results for slabs supported by 356 x 171 x 67 UBs

	Simply supported	**Not composite**		**Composite**	
	Value	**Value**	**factor**	**Value**	**factor**
Δ (m)	1.190E-03	1.40E-02	11.8	8.39E-03	7.05
Mx (kN m)	17.568	103.84	5.9	65.93	3.75
My (kN m)	31.752	29.94	0.94	29.84	0.91

The *factor* is the ratio of the value to the corresponding simply supported value (used here in preference to a % change because the differences are much larger than in Table 10.4). The Delta factor is a flexibility factor in that it is the amount by which the value is magnified by the flexibility of the beam supports

The simply supported model, used here as a reference model (Section 2.4.5), is the same as that used in Table 2.

Note that:

- The deflection and M_x moments of the beam supported models are much greater than the simply supported values.
- The main span of the beam supported models is in the X direction (as compared with the Y direction for the simply supported model)
- The My moments are not much affected by the beam supports
- The composite action reduces the deflection and the Mx moment significantly but only marginally affects the My moment.

The beam must be *very* stiff to model a simple support. It takes the largest beam in the UB range (914 x 419 x 388 kn/m) to come close to simulating such supports in this context.

Timoshenko and Woinosky-Krieger (1959) (Section 4, Table 48) give a solution for a square slab with corner supports and beams along the four sides. The solution uses the non-dimensional parameter:

$\gamma = EI/aD$ where EI is the stiffness of the beam, a represents the side length and D is the slab stiffness $D = Eh^3/(1-\nu^2)$.

The parameter γ may be used to characterise the interaction between the slab and the beam. From the solution in Timshenko (1959) it appears that the beams start to become effectively 'rigid' for $\gamma > 10.0$. Table 4 shows the γ values for the beam as used in the model of Figure 1(b) and for the stiffest universal beam in the tables. An average a

value of 7.0m was used. With the 914 UB the simply supported condition is still not being approached.

Table 4 γ values for beam supported slab

Beam section	γ value
356 x 171 x 67 UB	0.065
914 x 419 x 388 UB	2.40

Issues in relation to slab design

Table 4 gives a comparison of the assumptions for the conventional and the FE composite slab model of Figure 1(b). The slab will tend to behave under working load very differently from the design assumptions of the conventional method. Codes of practice tend to neglect the effect of the beam deflection for slab design. The code of practice methods can be justified on the basis of the lower bound theorem - a basic structural system which is capable of supporting the design loading is provided.

If the system is a conventional composite slab using ribbed sheeting as permanent formwork then the long span slab stiffness will be less than that of the short span resulting in less moment being taken by long span bending. Also any tendency to cracking because of neglect of the long span slab moments is unlikely to show in the sheeting.

Table 5 Slab model comparisons

Model	**short span**	**slab supports**	**long span**
Conventional	one-way	rigid vertically	Composite beams with slab contribution only over an equivalent slab width above the beams.
FE (Figure 1(b))	part of a two-way span	flexible beams	Slab takes axial force and moment over full width with significant bending moments at mid-span