Modern Structural Analysis - Introduction to Modelling

Supplementary information

Chapter 5 Section 5.12 Case study
Behaviour of a grillage frame

The cross references in the form 'n.m' are to sub-sections in *Modern Structural Analysis - Introduction to Modelling.*
The cross references with a single number are to items within this document.

The purpose of this case study is demonstrate some of the features of the behaviour of a grillage. It shows:

- How the relative bending stiffnesses of the cross beams to the main beams affect load transfer
- How the internal shears, moments and torques can be investigated.
- That the effect of torsional stiffness of the members is not a main factor in load distribution.

Figure 1 shows a plan view of the grillage

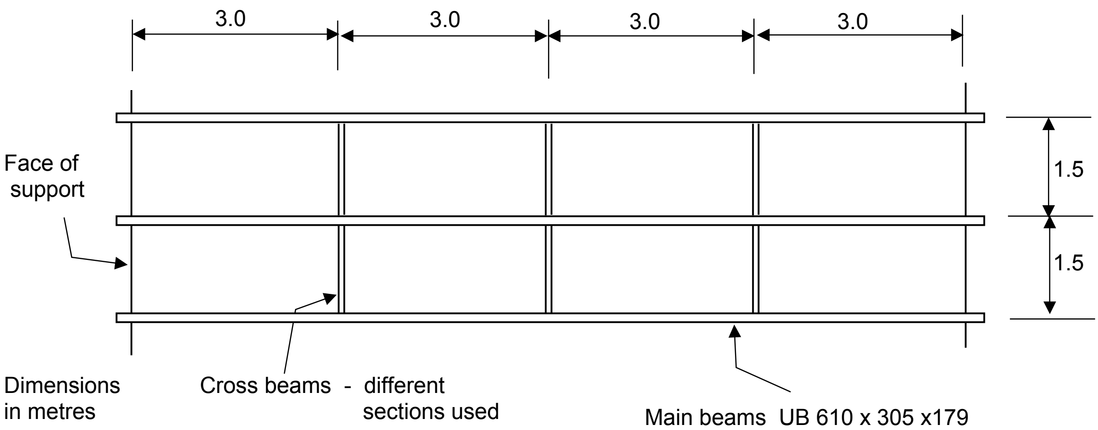


Figure 1 Plan view of grillage

The analysis model
Figure 2 shows a plan view of an analysis model of the grillage. Full bending and torsion continuity is assumed at the connections.

Elements LUSAS GRIL elements (3 degrees of freedom per node - Figure 5.15(b) are used in the model.

Element properties - See Table 1

Supports Vertical deformations at the nodes at the ends of the main beams (nodes 1,10,19,8,17,26) are restrained. Note that no allowance is made for 'landing' on the supports. The positions of these nodes should be inside the lines of the supports.

Loading A single checking load of 100kN is applied downwards at node 22.

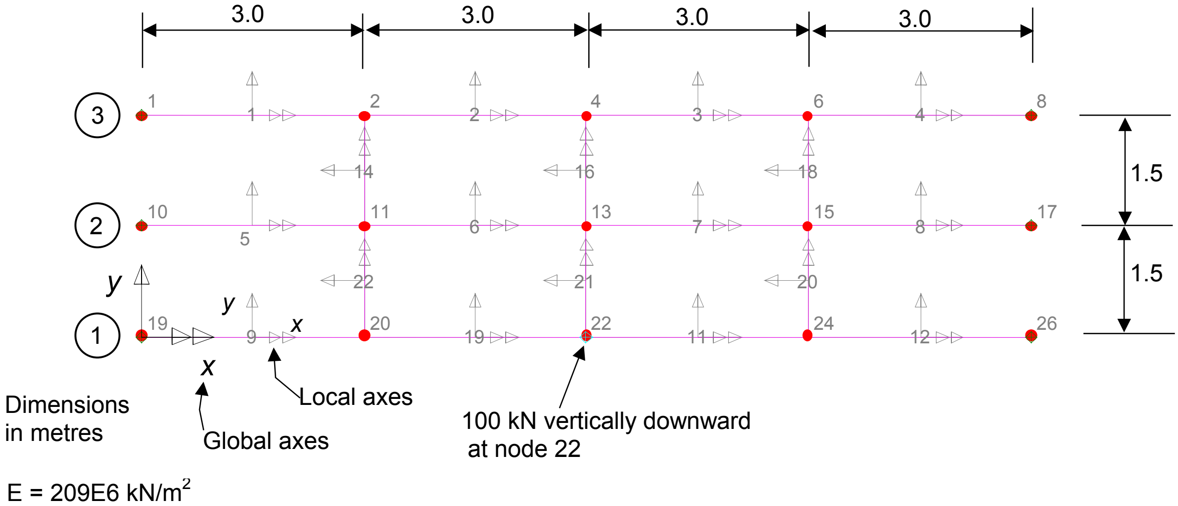


Figure 2 Plan view of analysis model of grillage

When establishing the data it is most important to ensure that the major axis of the elements is correctly oriented. In the case of the grillage shown in Figure 2 the I_y values quoted in Table 1 are the major axes values. They correspond to the local y axes of the members which are in the global xy plane for the main beams and the cross beams.

Sensitivity analysis - effect of the cross beam stiffness in spreading the load
The main function of the cross beams is to spread the load among the main beams. This sensitivity analysis shows how the relative stiffness of the cross beams to that of the main beams affects this load distribution.

Indicative parameters
Two *load transfer ratios* (LT) are considered:

1. SLT The proportion of the applied load of 100 kN which is transferred from the loaded beam to the other beams expressed as a percentage is denoted as the *System Load Transfer Ratio* - SLT - defined as follows:

   The load of 100.0 kN is applied to main beam 1 which has reactions at each end R_1. Main beams 2 and 3 have reactions R_2 and R_3 respectively. Therefore the load taken by main beams 2 and 3 is $2R_2 + 2R_3$. Therefore the proportion of the applied load taken by beams 2 and 3 expressed as a percentage is:

   $SLT = (2R_2 + 2R_3)/100*100$ (1)

   For overall vertical equilibrium of the system $100.0 = 2R_1 + 2R_2 + 2R_3$

   Hence substituting $2R_2 + 2R_3 = 100.0 - R_1$ into (1) gives:

   $SLT = (100 - 2R_1)$ (2)

2. *LLT* The proportion of the applied load which is transferred at its point of application is denoted as a *Local Load Trasfer Ratio-* *LLT* . This is calculated as: The shear force in the cross beam (element 21) which connects to the loaded node is denoted as S_{21}. Therefore the proportion of the load transferred across the grillage directly at node 22 expressed as a percentage) is:

$$LLT = S_{21}/100*100 = S_{21}$$

Variation in parameters

The model was run with 5 different cross beam sizes as shown in Table 1. The *LT* values are compared with a stiffness ratio - *SR* - which is the ratio of the *I* value of the cross beams to that of the main beams i.e. with the parameter:

$$SR = I_{cross}/I_{main}$$

Results

The values of the *LT* ratios are quoted in Table 1 and plotted in Figure 3 Note how for *SR* values less than about 0.05 the *LT* values (especially *LLT*) decreases significantly. On the basis of the *SLT* value, the UB356x127x33 (*SR* = 0.054) appears to be a candidate choice for the cross beam for optimum stiffness. But on the basis of the *LLT* value it might be better to use a stiffer section. *SLT* may be the better parameter

Table 3 also shows the *LT* value for the situation where the cross beams are effectively rigid in bending - the 'Rigid in bending' entry in Table 1 The table also shows that using the same cross section for the main and cross beams (*SR* = 1.0) is close to the fully rigid situation but that there is no point in making the cross beams as stiff as this

LLT increases monotonically with increase in *SR* but *SLT* increases with increasing SR and then starts to decrease at a value of *SR* = 0.2. The latter behaviour appears to be due to the way that elements 20 and 22 attracts shear force.

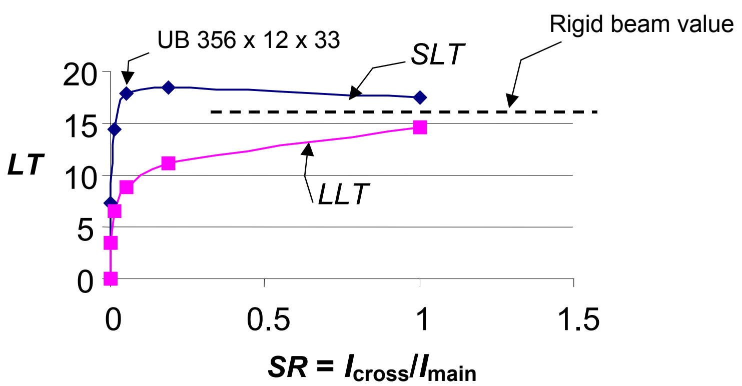


Figure 3 Load transfer in relation to relative beam stiffness

Table1 Model properties and results

Cross beam section	***I_y* (bending) m^4**	***I_x* (torsion) m^4**	***SR* = I_{cross}/I_{main}**	***R_1* kN**	***SLT* %**	***LLT* %**
Rigid in bending	1.53E-01		100	41.659	16.682	16.84
610x305x179*	1.53E-03	3.40E-6	1.0	41.249	17.502	14.600
457x152x67	2.89E-04	4.77E-7	0.1891	40.795	18.41	11.122
356x127x33	8.25E-05	8.79E-8	0.0539	41.062	17.88	8.884
203x102x23	2.11E-05	7.02E-8	0.0138	42.805	14.39	6.614
127x76x13	4.73E-06	2.85E-8	0.0031	46.312	7.376	3.524
No cross beam	0		0.00	50	0	0

* Main beam properties
All sections Universal Beams (ÚB)

Distribution of shear forces

Figure 4(a) shows the shear forces on a symmetric half of the grillage. The sign convention is shown in Figure 4(b). The local axes for the elements are shown on Figure 2.

Figure 4(a) shows the how the shear forces distribute across the central set of cross beams i.e. on line 22-13-14. Note how element 21 pushes up on node 22 to give it support. That element 16 is pushing up on node 14 indicates that the main beam 3 will tend to deflects upwards (it does).
Figure 5(b) shows the vertical forces which are applied to the cross beam - the forces which the cross beam applies to the main beams are equal and opposite.
Figure 5(c) is the shear force diagram using the same sign convention as for the shear force values in Figure 4.

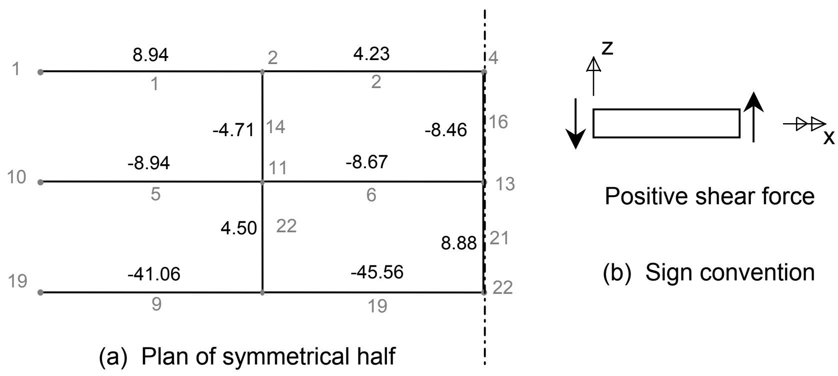
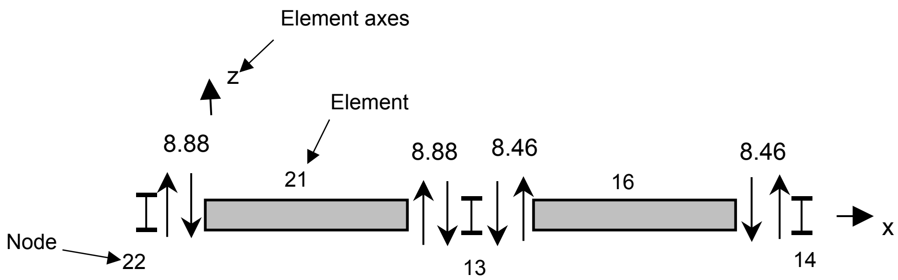
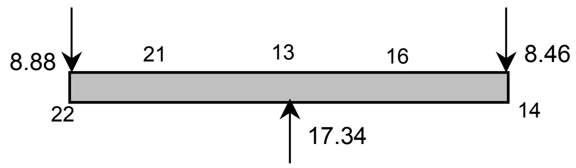


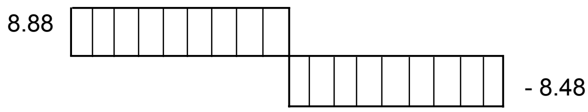
Figure 4 Shear forces in grillage elements



(a) Vertical forces on the ends of the cross beams and on the main beams



(b) Vertical forces on the cross beam from the main beam



(c) Shear force diagram

Figure 5 Vertical forces in beam 22-13-14

Distribution of moments

The moments on the element ends and on the nodes for cross beam 22-13-4 are shown in Figure 6(a). The moment on node 22 from element 21 is resisted by the main beam 1 as a torque. The value is low (25.05 - 24.83 = 0.22) indicating that the torsional stiffness of the main beam is not significant. (This can be checked by running the model with close to zero values for the torsional constants of the elements and comparing the results with torsion included).

The moment at node 14 from element 16 is also low due to the low torsional stiffness of the main beams. The difference between the end moments at node 13 i.e. 25.05-24.83 = 0.22 is transmitted as a torque to the central main beam.

Figure 6(b) shows the bending moment diagram for the cross beam. This diagram uses the structural engineering convention of 'drawn on the tension side' of the member.

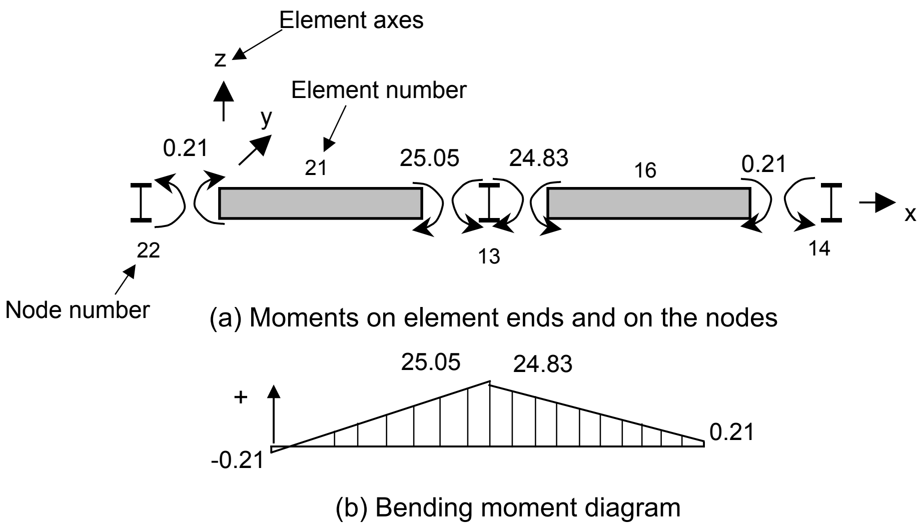


Figure 6 Moments in beam 22-13-14